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**TERMINAL VELOCITIES OF
WINDOW DIPOLES USED
IN HIGH ALTITUDE
WIND MEASUREMENTS**

by

T. W. G. Dawson, M.A., A.Inst.P.

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SUMMARY

Various theoretical formulae for the rate-of-fall of window dipoles at altitudes up to 100 Km are shown to be in close agreement with the best available experimental data. At low altitudes velocity is determined mainly by conventional aerodynamic drag. At approximately 20 Km, Lamb's theory of viscous flow is required whilst at 50-70 Km, corrections must be applied for 'slip-flow'. Finally, above about 75 Km, 'free molecule flow' theory is needed to get good agreement with experiment.

For the most accurate measurements of wind structure it is desirable to employ dipoles with the smallest possible terminal velocity. This means that the dipoles should have low density and the smallest possible thickness.

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1 INTRODUCTION

One well-established technique¹⁻³ for the measurement of winds at high altitudes above the surface of the earth is to release into the atmosphere, by means of a rocket, a cloud of small strips of metal known either as 'window dipoles' or 'chaff'. These fall towards the earth under the action of gravity and are also blown to and fro by the wind. The dipoles are easily visible to a radar equipment and thus, provided the rate-of-fall is not too great, the magnitude and direction of high-altitude air-streams can be computed from range and angle data obtained by the radar. In a typical experiment a single cloud might be formed from one million dipoles with a total weight of one pound. This might be released at 85 Km altitude and would act as a wind-marker for a time of the order of 30 minutes before becoming too diffuse to be tracked by radar.

In a previous report⁴, a theory was developed for the rate-of-fall of dipoles released at various altitudes. At that time, however, the experimental data was rather inaccurate and it was only possible to check that the theory and experiment were in general agreement. Since then, the following developments have occurred:

- (i) much more precise measurements have been made⁵⁻⁷ of the rate-of-fall at various altitudes,
- (ii) more accurate measurements have been made of the density, pressure and temperature of the upper atmosphere⁸,
- (iii) another report⁹, containing theoretical formulae for rate-of-fall, has become available.

The purpose, therefore, of this present report is to review the theory (Section 2) and then to make an accurate comparison between theory and experiment in order to discover any discrepancies and to determine how to make the dipoles fall as slowly as possible.

2 THEORETICAL FORMULAE

2.1 General theory

The general equation of motion for a dipole falling through still air is

$$m \cdot \frac{dv}{dt} = mg - \frac{1}{2} \rho v^2 A C_D \quad . \quad (1)$$

In this equation the LHS represents mass times acceleration whilst the RHS is the gravitational force less aerodynamic drag. Buoyancy forces are assumed zero⁵. Also:

V = downward velocity

ρ = air density

A = dipole projected area

m = dipole mass

C_D = the drag coefficient (defined by equation (1)).

To find the dipole terminal velocity, set $dV/dt = 0$:

$$V^2 = \frac{2mg}{\rho A C_D} \quad . \quad (2)$$

Thus to obtain V it is necessary to calculate the drag coefficient

In general¹⁰, C_D is a function of Reynolds number R and Mach number M :

$$R = \frac{V \rho w}{\mu} \quad (3)$$

$$M = \frac{V}{c} \quad (4)$$

where w = dipole width

μ = viscosity of air

c = velocity of sound.

The Reynolds number may be written in terms of any characteristic dimension of the body. In this report, however, width is chosen because it appears from the results given below that most dipoles tend to fall with their length in the horizontal plane and in this situation the various theories show that the width, rather than the thickness or length, is the most important dimension affecting C_D and V . The velocity of sound at any altitude may be computed from¹⁰

$$c^2 = \frac{\gamma P}{\rho} \quad (5)$$

where γ = ratio of specific heats of air molecules = 1.41

P = air pressure.

2.2 Low altitudes

At low altitudes, calculation of C_D is a straightforward matter; the falling dipole causes a disturbance in the surrounding air which acts as a continuous fluid. Near ground level it is found that

$$M \ll \sqrt{R}$$

and

$$R > 1 \quad (6)$$

and C_D may be found from standard text-books on low-speed aerodynamics.

If the main surface of the dipole is in the horizontal plane then experimental results can be approximated by the relations⁴:

$$C_D \sim 2 \quad \text{at } R > 20 \quad (7)$$

$$C_D \sim \frac{8.5}{\sqrt{R}} \quad \text{at } 20 > R > 2 \quad (8)$$

Taking $A = Lw$ where L is the dipole length and substituting for C_D in equation (2), it is then found

$$V^2 = \left(\frac{mg}{L} \right) \cdot \frac{1}{\rho w} \quad \text{at } R > 20 \quad (9)$$

$$V^3 = \left(\frac{mg}{L} \right)^2 \cdot \frac{1}{18 \rho w \mu} \quad \text{at } 20 > R > 2 \quad (10)$$

If a flat-plate has an edge at right-angles to the flow, (e.g. the long edge), then the drag components are skin friction and forebody drag. The latter contribution is, however, negligible for dipoles at low altitudes. The standard formula for skin friction in laminar flow conditions is that due to Blasius¹¹:

$$C_D = \frac{1.328}{\sqrt{R}}$$

For very small M and low R , however, Janour¹² has shown that the actual drag coefficient is approximately 1.5 times larger than that given by Blasius. Hence for skin friction on two sides of the dipole:

$$C_D \sim 1.5 \times 2 \left(\frac{1.328}{\sqrt{R}} \right) \quad \text{for } M < 0.2 \quad (11)$$

50 > R > 2

where R is based on the dipole dimension which is parallel to the downward motion. It follows from equation (2):

$$V^3 \sim \left(\frac{mg}{L} \right)^2 \cdot \frac{1}{4 \rho w \mu} \quad M < 0.2 \quad (12)$$

50 > R > 2

If the dipole falls with its length vertical instead of width vertical then w and L must be interchanged in the above equation.

2.3 20 Km altitude

At approximately 20 Km altitude it is found, for the dipoles discussed in this report, that the Reynolds number falls below unity. Such a low number is outside the province of 'aircraft' aerodynamics but was considered by Lamb¹³ in his theory of viscous flow. By 'viscous' flow is meant flow in which a boundary layer of air molecules is permanently attached to the dipole and in which the mean free path of air molecules is small compared with any dipole dimension. The flow is assumed laminar and the dipole velocity is relatively low ($R \leq 1$ and $M/R \ll 1$).

Lamb's theory applies to a cylinder falling with its length horizontal and unfortunately C_D is obtained in a form which contains $\log R$ and does not permit a simple solution of equation (2) for the velocity. However an approximation to Lamb's equation, correct to 20%, is¹⁴:

$$C_D \sim \frac{10}{R^{0.8}} \quad \text{at } 0.001 < R < 2 \quad (13)$$

The above formula is for a cylinder, with R in terms of the cylinder diameter (d). Birstow¹⁴, however, has shown that a flat strip width w falling with its length horizontal has the same motion as the cylinder provided

$$d = 0.83w \quad (\text{strip broadside-on to flow}) \quad (14)$$

$$d = 0.3w \quad (\text{strip edge-on to flow}) \quad (15)$$

Substituting in (2) it is then found

$$V^6 = \left(\frac{mg}{L}\right)^5 \cdot \frac{1}{5^5 \mu^4 k \rho w} \quad \text{for } 2 > R > 0.001 \quad (16)$$

and $\frac{M}{R} < 0.01$

where $k = 0.83$ or 0.3 according to whether the strip is broadside-on or edge-on to the flow.

2.4 Slip-flow region

As the altitude is further increased the mean free path of air molecules becomes a significant fraction (1-10%) of the boundary layer thickness and a phenomenon known as 'slip' occurs: the layer of gas adjacent to the dipole begins to slide at a finite velocity along the dipole surface. For $R < 1$, the effect is believed to occur¹⁰ at:

$$0.01 < \frac{M}{R} < 0.1 \quad . \quad (17)$$

The only available formula for C_D in this region, with $R < 1$, appears to be that due to Tsien¹⁵ for a horizontal cylinder:

$$C_D = \frac{8\pi}{R \cdot \left(\log_e \left(\frac{8}{R} \right) - 1.28 + 3.55 \frac{M}{R} \right)} \quad . \quad (18)$$

This is a modification of Lamb's formula* for finite M/R . Equation (18) is actually Tsien's formula rearranged (i) so that R is based on the cylinder diameter instead of cylinder radius as assumed by Tsien, and (ii) for the special case where $\gamma = 1.41$ and where air molecules are assumed to reflect in a diffuse way¹⁰ on striking the dipole. The formula does not permit an explicit solution for V .

The simplest way of finding V appears to be that suggested by Barr⁹: by substituting for R and C_D in equation (2) it is found**:

$$\rho = \frac{\left(\frac{4\pi L \mu^2}{mg} \cdot R - \frac{2.98 \mu}{c} \right)}{d(0.8 - \log_e R)} \quad . \quad (19)$$

* Lamb's equation is similar to equation (18) for $M/R = 0$ except that Lamb has $(\gamma - 2)$ in place of 1.28.

** The equation given in Barr's report omits the R shown in the numerator of (19).

Thus assuming μ and c are constant, it is possible to find the density ρ which corresponds to a given R . Once ρ and R are known, it follows from the definition of R that:

$$V = \frac{\mu}{d} \cdot \frac{R}{\rho} \quad (20)$$

whilst the altitude corresponding to V may be deduced from altitude-density tables (Table 1).

For a flat strip, it is assumed equations (14) and (15) still apply.

2.5 Free molecule flow region

At greater altitudes, (above 70 Km) the mean free path of air molecules becomes much greater than the dipole dimensions (Table 2). In this case there is no boundary layer and molecules reflected or re-emitted from the dipole surface do not collide with air molecules until they have travelled a considerable distance, i.e. the dipole does not cause any significant distortion of the air stream. This situation is termed¹⁰ 'free molecule flow' and drag is determined solely by molecule-surface interaction, being independent of air viscosity. It has been shown theoretically that free molecule flow should occur at

$$M > 3R \quad . \quad (21)$$

It will be assumed¹⁰ that molecules are reflected in a diffuse manner and that the dipole temperature is equal to the air temperature. It is then found¹⁰ that for a cylinder falling with its axis horizontal:

$$C_D = \frac{\pi^{0.5} e^{-0.5s^2}}{s} [(s^2 + 1.5) I_0(0.5 s^2) + (s^2 + 0.5) I_1(0.5 s^2)] + \frac{\pi^{1.5}}{4s} \quad (22)$$

where I_0 and I_1 are the modified Bessel functions. At small s , $I_0 \sim 1$ and $I_1 \sim 0$. The quantity s , known as the 'molecular speed ratio' is

$$s = \frac{V}{C_m} \quad (23)$$

where C_m is the most probable molecular velocity¹⁰:

$$C_m^2 = \frac{2R'T}{W} \quad (24)$$

where R' is the gas constant, W the molecular weight of air and T the temperature.

In the preliminary calculations it was noticed that for window dipoles below 100 Km,

$$s < \frac{1}{2} \quad (25)$$

and in this special case equation (22) simplified to

$$C_D \sim \frac{4}{s} \quad . \quad (26)$$

Barr⁹ has also considered the case of the cylinder moving with its axis at right angles to the flow: working from an expression for C_D which is more general than that given in equation (22) he found:

$$C_D = \frac{1.5 \pi^{0.5} + 0.25 \pi^{1.5}}{s} \quad \text{for } s \leq 0.4 \quad . \quad (27)$$

Evaluating the numerator, this is found to be

$$C_D = \frac{4.05}{s} \quad (28)$$

in close agreement with (26).

Turning to the flat plate, it is found¹⁰:

$$C_D = \frac{2}{\pi^{0.5} s} \left[e^{-(s \sin \theta)^2} + \pi^{0.5} s \sin \theta \left(1 + \frac{1}{2s^2} \right) \operatorname{erf}(s \sin \theta) + \pi \sin^2 \theta \right] \quad . \quad (29)$$

This equation is based on the same assumptions as (22). θ is the angle of attack, i.e. the angle between the surface and the flow direction (vertical). $\operatorname{erf}(s \sin \theta)$ is the 'error function' and at small $s \sin \theta$:

$$\operatorname{erf}(s \sin \theta) \sim \frac{2}{\pi^{0.5}} \cdot s \sin \theta \quad . \quad (30)$$

At small ($s \sin \theta$), equation (29) simplifies to:

$$C_D = \frac{2}{\pi 0.5} (1 + 4.142 \sin^2 \theta) . \quad (31)$$

Hence for a dipole edge-on and length either horizontal or vertical

$$C_D = \frac{1.13}{s} \quad (32)$$

and for a dipole broadside-on

$$C_D = \frac{5.8}{s} . \quad (33)$$

Also if dipoles are in random orientations, i.e. the probability of any value of θ is proportional to $\sin \theta$, then the mean drag coefficient obtained by integration of (31) over all θ is

$$C_D = \frac{4.25}{s} \quad (34)$$

which is nearly the same as the value for the horizontal cylinder (equation (26)).

As in the previous cases, the terminal velocity may be found by substitution of C_D in equation (2). For example (26) yields:

$$V = \left(\frac{mg}{L} \right) \cdot \frac{1}{2 \rho w C_m} \quad (35)$$

whilst for the flat dipole broadside-on to the air flow (33)

$$V = \left(\frac{mg}{L} \right) \cdot \frac{1}{2.9 \rho w C_m} . \quad (36)$$

3 LOW ALTITUDE MEASUREMENTS

Although a number of measurements have been made of the rate of fall of window dipoles in the lower atmosphere, many of the data are unfortunately rather inaccurate and cannot be used for a precise check of the theoretical formulae. Recently, however, Jiusto and Eadie⁵ have made a number of careful measurements in a high-altitude test-chamber, simulating altitudes up to 20 Km. Their results, together with details of the dipoles, are reproduced in Fig.1. The procedure was to release a number of dipoles and then note, at each simulated altitude, the times taken by the fastest and slowest dipoles to fall a standard distance (2.5 metres). A record was also kept of the median time.

Thus the three curves in Fig.1 show the maximum, median and minimum velocity as a function of altitude.

Also plotted on Fig.1 are points calculated with the aid of equations (10), (12) and (16). The parameters assumed for the atmosphere are set out in Table 1. It can be seen that there is good agreement between theory and experiment: the slope of the theoretical curves are almost identical with the measured values whilst the ratio of maximum to minimum velocity found to be 6.8 from the experimental data compares well with the theoretical values of 7.0 at 0 Km and 7.4 at 20 Km.

There is probably little significance in the coincidence of the median velocity curve with the theoretical points for edge-on horizontal dipoles. It is commonly observed near ground level that most dipoles fall with their length and width horizontal so presumably the median resulted from a few dipoles falling vertically together with a larger number falling broadside-on. Nevertheless it is useful to note that the median result fits the edge-on formula.

The fact that the observed maximum velocities are slightly less than the theoretical values might be explained by assuming no dipoles fall the whole distance with their lengths exactly vertical. As just mentioned the most stable dipole orientation appears to be that in which the length is horizontal and consequently vertical dipoles might tend to turn and so have reduced velocities.

The fact that no dipoles fall quite as slowly as the theoretical minimum is perhaps rather more puzzling; at 0 Km the difference between theory and experiment is approximately 15%. The authors do not give the estimated accuracy of the measurements but it can be observed that a dipole falling at 0.35 m/sec takes 7 seconds to fall 2.5 metres and hence assuming a timing to an accuracy of 0.2 second, the experimental error was probably no greater than 3%, i.e. only one fifth of the discrepancy. Possibly no dipole fell the whole distance with its surface always horizontal.

The authors state that the experimental results were appropriately corrected for the time required for the chaff to reach terminal velocity but no details are given of the magnitude of the correction.

Measurements were also made with S-band dipoles. These had the same width and thickness as those in Fig.1 but were 3.34 times longer. It was found that the rates of fall for the longer dipoles were the same as those in Fig.1; in agreement with equations (9), (10) and (12) which all state that V is proportional only to the mass per unit length.

4 HIGH ALTITUDE MEASUREMENTS

The most comprehensive series of high altitude measurements of rate-of-fall appear to be those reported by Smith⁶. A total of 742 rate-of-fall versus altitude measurements were made at altitudes in the range 40-84 Km above Nevada, the Marshall Islands, Johnston Island and White Sands. In each case the rate-of-fall was measured over a one-minute interval. The curve which best fitted all experimental results was found to be:

$$\log_{10} f = 5.014 - 2.847 \times 10^{-5} z + 9.294 \times 10^{-11} z^2 \quad (37)$$

where f = rate of fall is ft/minute

z = altitude in ft.

This relation is shown as a thick curve in Fig.2. Also in Fig.2 are set out details of the dipoles used and a set of four vertical lines which indicate the standard deviation in individual results at various altitudes. Smith actually gives more details of standard deviations but only four representative values are included in Fig.2 so as to indicate the order of magnitude without obscuring other detail.

Some of the Johnston Island measurements discussed by Smith were also analysed in greater detail by Rapp⁷. His mean curve, for altitudes between 72.5 and 80 Km is included in Fig.2 and can be seen to be in close agreement with equation (37).

As the dipoles used in the high altitude measurements were not identical with those used at low altitudes (Section 3), the results in Section 3 could not be included, as they stood, in Fig.2. In view, however, of the good agreement between theory and experiment at low altitudes it was felt useful to scale-up the median low-altitude results, with the aid of equation (12) and plot these also on Fig.2. The resulting curve is shown as a thick line between 0 and 20 Km. An interpolation has been made in Fig.2 between the low altitude results (below 20 Km) and the high altitude results (above 40 Km) in order to gain some ideas of the likely variation in V at all altitudes below 83 Km.

Also drawn on Fig.2 are the theoretical curves for the slowest falling dipoles (w and L in a horizontal plane). Each curve is drawn as a solid line for those altitudes at which R and M/R are within the theoretical boundary conditions set out in Section 2 and as a dashed line at other altitudes. The 'free-molecule theory' curve is not drawn above 90 Km altitude because the molecular speed ratio s then exceeds the value 0.5 assumed in the approximate

equations (32)-(36). In the free-molecule zone, a second theoretical curve (dotted and labelled 'MAX') has been drawn-in for the fastest falling dipoles (equation (32)).

Comparing theory and experiment it may be observed that at each altitude there is remarkably good agreement between the slopes of the experimental curve and the slope of the appropriate theoretical curve. The four theoretical curves form a continuous S-shape very similar to the experimental result.

In Fig.2 it can be seen that the experimental curve takes a fairly sharp bend upwards, i.e. the slope departs from that predicted by the viscous drag formula, at an altitude of about 65 Km. This is almost exactly the altitude at which the mean free path of air molecules becomes equal to the dipole width (Table 2). Here the ratio $M/R \sim 0.7$. At higher altitudes the slip-flow formula appears to be in reasonable agreement with experiment at least until $M/R > 3$ (75-80 Km) where the free-molecule formula applies. (Incidentally the slip flow formula (19) cannot be applied at very low Reynolds numbers because the numerator becomes negative.)

One unexpected result is that the viscous drag and slip-flow formulae both seem to apply at higher altitudes than predicted. According to theory¹⁰, slip flow should commence and the viscous drag formula should begin to fail at $M/R \sim 0.01$, (i.e. 35 Km) c.f. the experimental value, $M/R \sim 0.7$ (65 Km). Further, whereas the slip-flow regime is normally considered to end at $M/R \sim 0.1$ (50 Km), in practice the equation appears reasonably accurate up to $M/R \sim 3$ (75 Km). In other words significant changes in the drag coefficient, due to finite molecular mean free path, appear to occur only at much larger mean free paths than estimated theoretically.

A further point of interest is that in a discussion of the theory, Emmons¹⁰ (1958) states (i) that no experimental results are available to check the slip-flow formula for the drag on a cylinder (equation (18)) and (ii), in the region

$$0.1 < \frac{M}{R} < 3$$

called the 'transition regime', very little is known about C_D either theoretically or from experiment*. The present results suggest that equation (18)

* Some laboratory measurements have been made¹⁶ in the transition regime for large Mach numbers ($M > 2$) but apparently none have been made for the low Mach numbers encountered in the present study.

is in fact correct but that the slip-flow region extends right up to the free-molecule zone, i.e. there is no need to define a separate 'transition regime'.

The theoretical curves in Fig.2 are for the slowest falling dipoles. Presumably the median experimental results are approximately 1.5 times greater because many dipoles tend to fall edge-on rather than broadside-on. At the higher altitudes the curves for minimum velocity and for maximum velocity run at approximately 1.5 standard deviations either side of the mean experimental curve: taking into account the fact that at the highest altitude the error in a single measurement of rate-of-fall may be of the order of 10 m/sec (Rapp⁷), it is probable that most of the scatter in the experimental data arose from measurement errors and there is, therefore, no strong evidence that any dipoles fell with velocities outside the theoretical limits.

One assumption made in the foregoing discussion is that the measurements were all made on dipoles falling at the terminal velocity. This point was discussed by Rapp⁷ and was also considered, for the free molecule region, by Barr⁹. Both conclude that the assumption is very probably correct.

At low altitudes it was found that the median results agreed closely with the theoretical values for horizontal, edge-on dipoles. To see if this agreement also held at greater altitudes, the experimental results in Fig.2 were reproduced in Fig.3 together with the theoretical curves for dipoles falling with the width vertical. (The free-molecule curve was obtained by fitting to the slip-flow curve at the point where $M/R = 3$ and corresponds to $C_D \sim 2.6/s.$)

Fig.3 shows that the 'edge-on' theoretical curves are in close agreement with experiment and it appears that the relation: (median velocity) \sim (theoretical velocity for horizontal, edge-on dipoles) holds at all altitudes investigated.

5 DISCUSSION

5.1 Equation for rate-of-fall

It will have been noticed that the theory outlined in Section 2 is complicated and the individual equations, particularly those for slip-flow, are tedious to evaluate. No less than four separate theories are employed and unfortunately, until numerical values of V and R and M have been worked-out for a given altitude it is not possible to decide whether the particular equation used was in fact the correct one. It would clearly be useful to have an approximate formula valid over the whole range of altitude.

To obtain such an approximate equation it may be noticed that the viscous drag theory is only slightly in error at all altitudes up to that at which there are pronounced slip effects. Near ground level the error in the viscous drag theory is small and would be less with thinner dipoles (smaller R). Also, once slip-flow effects have become important, only a small increase in altitude is required before entering the free molecule theory domain. Thus an approximate theory might be based solely on viscous drag and free molecule theories.

One way of combining the two theories is simply to add the two velocities together:

$$V = 110 \left(\frac{mg}{L} \right)^{5/6} \cdot \frac{1}{(\rho w)^{1/6}} + 2 \times 10^{-5} \left(\frac{mg}{L} \right) \cdot \frac{1}{\rho w} \quad . \quad (38)$$

Here the first term on the right hand side is the viscous drag term (equation (16)) with μ set at the mean value 1.5×10^{-4} cgs units, whilst the second term is the free molecule equation (31) with $C_m = 4 \times 10^4$ and $C_D = 2.6/s$ (Section 4). At low altitudes the second term is negligible and conversely at high altitudes the first term can be ignored.

Substituting for g in equation (38) and re-arranging into a more convenient form:

$$V = \frac{35000 m_o}{6 \sqrt{m_o} \rho w} + \frac{0.02 m_o}{\rho w} \quad (39)$$

where V = terminal velocity - cms/second

m_o = mass per unit length ($m_o = m/L$) gms/cm

w = dipole width (cm)

ρ = atmospheric density in cgs units (Table 1).

This equation is the approximation for V for all altitudes below 100 Km. Points calculated from (39) are shown as small crosses in Fig.3 and can be seen to lie close to the experimental curve, the error being less than the standard deviations in the measurements (Fig.2).

5.2 Slowest-falling dipoles

In measurements of wind intensity and direction, it is desirable to use slow-falling dipoles for two separate reasons,

(i) to reveal detail in the wind structure, e.g. measurements with fast--falling copper dipoles revealed considerably less detail than those with slower-falling nylon dipoles³,

(ii) to extend the useful lifetime of each window cloud.

At the lower altitudes, it is clear from equation (39) or from the theory in Section 2,3 that apart from very slight variations due to the sixth root term, the terminal velocity varies only with the mass per unit length:

$$V \propto m_o \quad . \quad (40)$$

Some types of chaff mentioned in the literature^{5,6,9} are as follows:

| Type | m gms | L cms | m_o |
|---------------------------|------------------|------------|------------------|
| | $\times 10^{-4}$ | | $\times 10^{-4}$ |
| aluminium strip (Fig.1) | 0.95 | 1.52 | 0.62 |
| aluminium strip (Fig.2)* | 7.5 | 5.0 | 1.5 |
| nylon cylinder (0.0035") | 8.1 | 4.8 | 1.7 |
| nylon cylinder (0.008") | 19.5 | 4.8 | 4.1 |
| nylon cylinder (0.012")* | 47.6 | 4.8 | 10 |
| copper cylinder (0.010")* | 216 | 4.8 | 45 |

(* denotes types reported used in atmospheric wind measurements)

It can be seen that the 0.012 inch nylon used in wind measurements was relatively fast-falling. The strips or thinner nylon would have been preferable.

At the highest altitudes the drag coefficient is approximately the same for strips and cylindrical dipoles (equations (34) and (26)) and

$$V \propto \frac{m_o}{w} \quad \text{or} \quad \frac{m_o}{d} \quad . \quad (41)$$

Thus for a strip where $m_o = \rho_m \cdot wt$:

$$V \propto \rho_m \cdot t \quad (42)$$

where ρ_m is the density of the dipole material.

Similarly for a cylindrical dipole where $m_o = \rho_m \cdot \pi d^2/4$

$$V \propto \rho_m \cdot \pi d/4 \quad . \quad (43)$$

It follows that for minimum rate-of-fall at high altitude, the material density should be kept low and also strips are probably preferable to cylinders because the thickness (t) of a strip can probably be made less than the diameter (d) of a metal or nylon thread. Typical values are $t = 1.14 \times 10^{-3}$ cms (strip dipoles in Fig.1) and $d = 8.89 \times 10^{-3}$ cms, (thinnest nylon dipoles discussed by Barr⁹).

6 CONCLUSIONS

- (1) Measurements of the rate-of-fall of window dipoles at high altitudes appear to be in good agreement with the theory. At low altitudes viscous drag effects are dominant whilst above 75 Km, free-molecule theory applies. In the intermediate 'slip-flow' region, the experimental results appear to confirm Tsien's equation for the drag on a cylinder.
- (2) At low altitudes (e.g. below 50 Km) the slowest falling dipoles are those which have lowest mass per unit length. At high altitudes, (e.g. above 70 Km) the terminal velocity is proportional to the density and thickness of the metallic strip.
- (3) In view of the possible number of ways of falling the agreement between the theory and practice examined in this report might seem to be fictitious; there is no evidence available to show what would happen for dipoles of different geometries or at radically different altitudes.

Table 1

ATMOSPHERIC PRESSURE, DENSITY, VISCOSITY AND TEMPERATURE

All quantities are expressed in cgs units

| Altitude Km | Pressure P | Density ρ | Viscosity μ | Temperature T $^{\circ}\text{K}$ |
|-------------|-------------------|----------------------|----------------------|----------------------------------|
| 0 | 1×10^6 | 1.2×10^{-3} | 1.8×10^{-4} | 288 |
| 10 | 2.6×10^5 | 4×10^{-4} | 1.4×10^{-4} | 223 |
| 20 | 5.4×10^4 | 9×10^{-5} | 1.4×10^{-4} | 217 |
| 30 | 1.2×10^4 | 2×10^{-5} | 1.5×10^{-4} | 227 |
| 40 | 2.9×10^3 | 4×10^{-6} | 1.7×10^{-4} | 250 |
| 50 | 800 | 1×10^{-6} | 1.7×10^{-4} | 270 |
| 60 | 210 | 3×10^{-7} | 1.7×10^{-4} | 250 |
| 70 | 54 | 9×10^{-8} | 1.4×10^{-4} | 220 |
| 80 | 10 | 2×10^{-8} | 1.2×10^{-4} | 180 |
| 90 | 1.6 | 3.2×10^{-9} | 1.2×10^{-4} | 180 |
| 100 | 0.29 | 5×10^{-10} | - | 210 |

Table 2APPROXIMATE REYNOLDS NUMBERS, MACH NUMBERS AND MEAN FREE PATHS

The numbers are calculated from the data in Table 1 and the observed median velocities (Fig.2). The Reynolds numbers are based on the dipole width.

| Altitude Km | R | M | M/R | mean free path cms |
|----------------|-------|-------|--------|-----------------------|
| 0 | 20 | 0.002 | 0.0001 | 7×10^{-6} |
| 10 | 15 | 0.004 | 0.0003 | 2×10^{-5} |
| 20 | 5 | 0.007 | 0.0014 | 9×10^{-5} |
| 30 | 1.5 | 0.009 | 0.006 | 4.4×10^{-4} |
| 40 | 0.3 | 0.01 | 0.03 | 0.002 |
| 50 | 0.1 | 0.012 | 0.1 | 0.008 |
| 60 | 0.04 | 0.017 | 0.5 | 0.026 |
| 70 | 0.025 | 0.03 | 1.4 | 0.093 |
| 80 | 0.025 | 0.13 | 5 | 0.41 |
| 90 | 0.02 | 0.7 | 30 | 2.5 |
| 100 | 0.02 | 3.5 | 200 | 16 |

SYMBOLS

A dipole projected area normal to airflow (sq cms)
 c velocity of sound (equation (5) - approximately 3×10^4 cms/sec)
 C_D drag coefficient
 C_m most probable molecular velocity (equation (24) - approximately $(4 \times 10^4$ cms/sec)
 d diameter of cylindrical dipole (cms)
 f rate of fall in units of feet per minute
 g acceleration due to gravity (981 cms/sec² at ground level, 957 cms/sec² at 80 Km)
 L dipole length (cms)
 m dipole mass (gms)
 m_o dipole mass per unit (gms/cm)
 M Mach number (equation (4))
 P air pressure
 R Reynolds number (equation (3))
 R' gas constant
 s molecular speed ratio (equation (23))
 t dipole thickness (cms)
 T absolute temperature (Table 1)
 V terminal fall velocity through still air (cms per sec)
 w dipole width (cms)
 W molecular weight of air
 Z altitude in feet
 γ ratio of specific heats of air molecules (1.41)
 ρ air density (Table 1)
 ρ_m mean density of dipole material
 μ air viscosity (Table 1)

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| Dipoles | |
|------------------------|---------------------------|
| LENGTH | 1.52 cms |
| WIDTH | 0.020 cms |
| THICKNESS | 0.00114 cms |
| MASS* | 9.45×10^{-5} gms |
| * ASSUMING S.G. = 2.67 | |

THE POINTS ARE THE THEORETICAL VALUES FOR DIPOLES FALLING IN THE ORIENTATIONS SHOWN IN THE PICTURES.

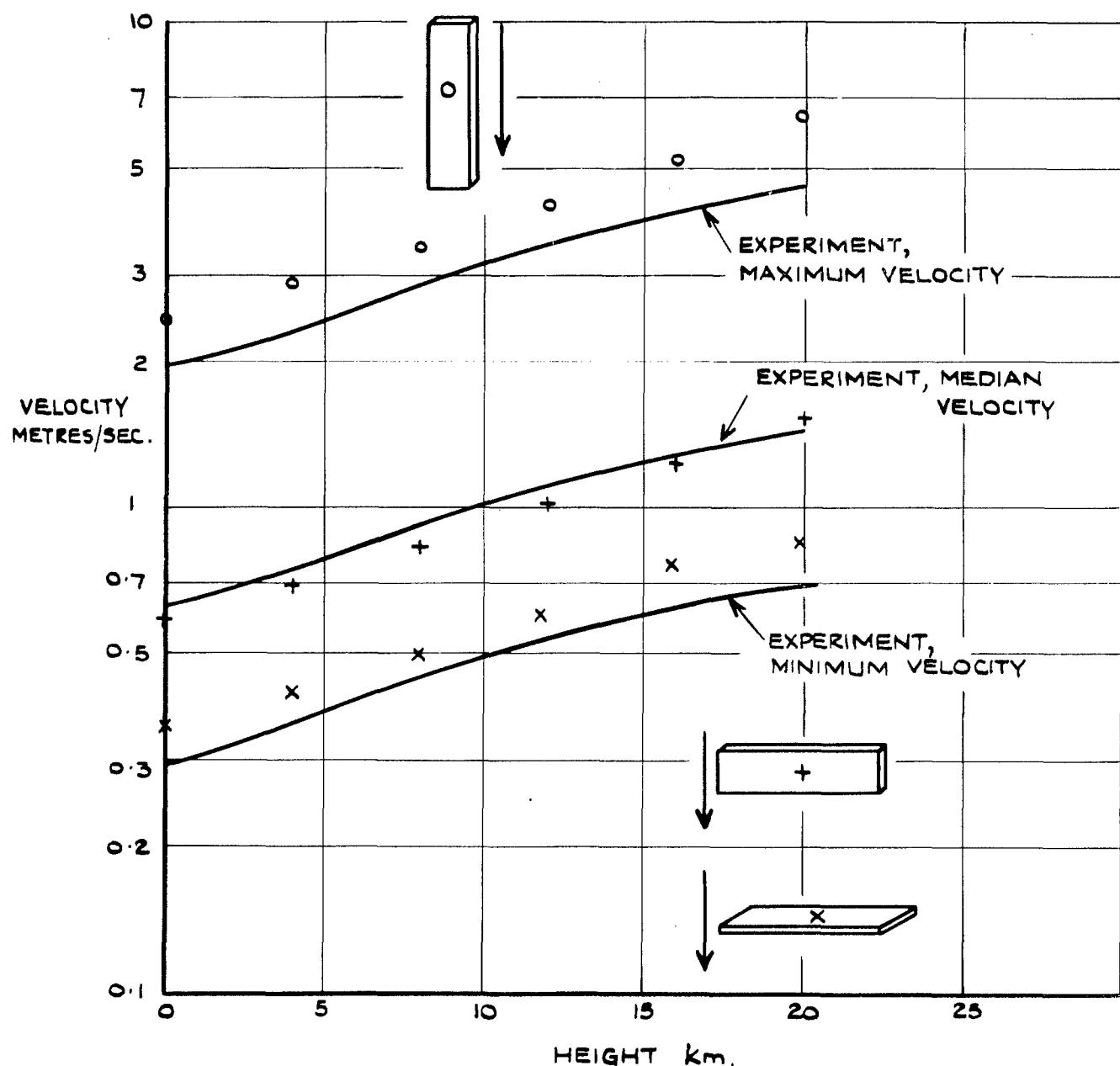


FIG. 1. VELOCITIES AT LOW ALTITUDES

Fig. 2

RAD/P 6341

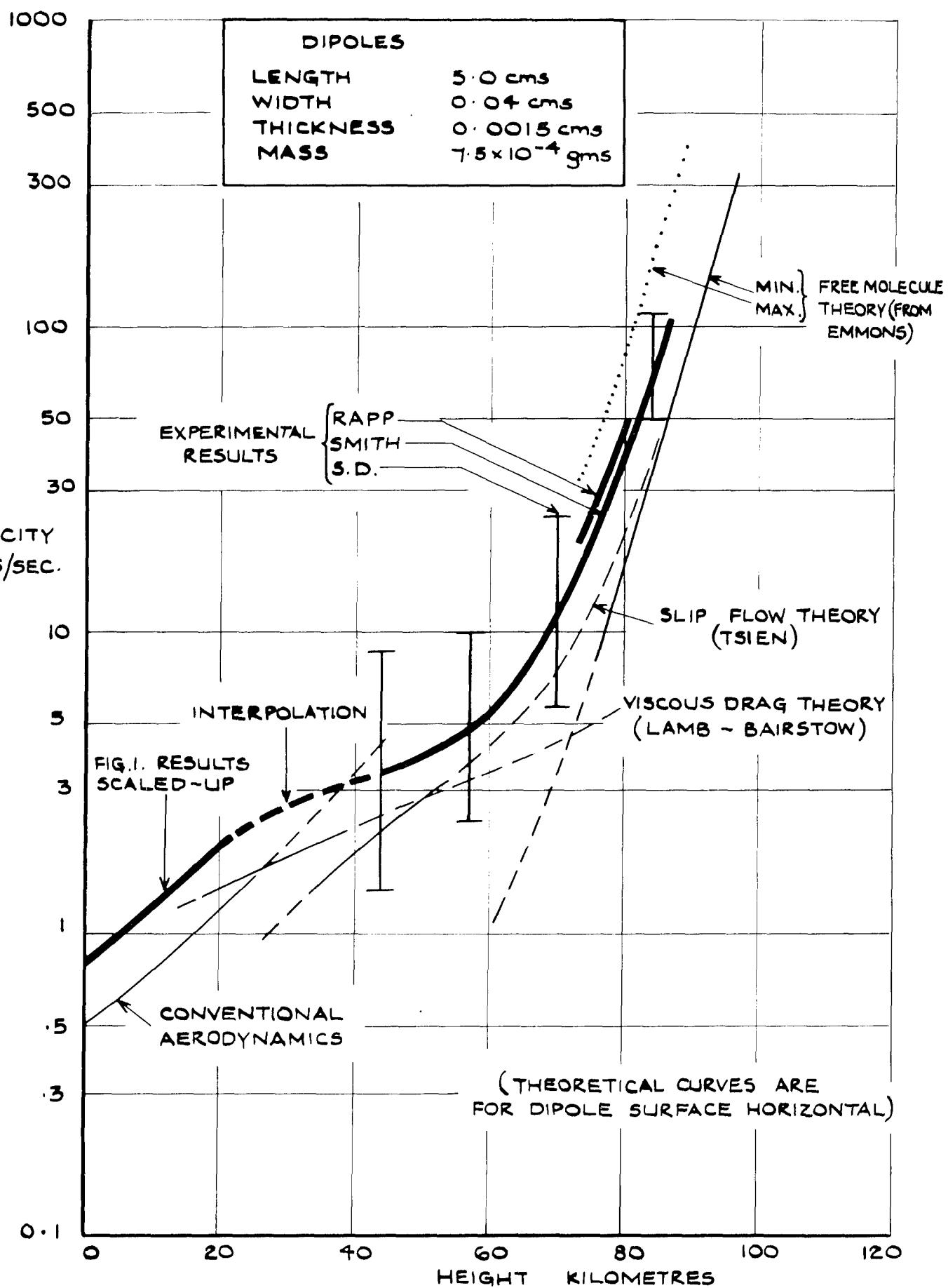


FIG. 2. VELOCITIES AT HIGH ALTITUDES - MINIMUM VELOCITY THEORY

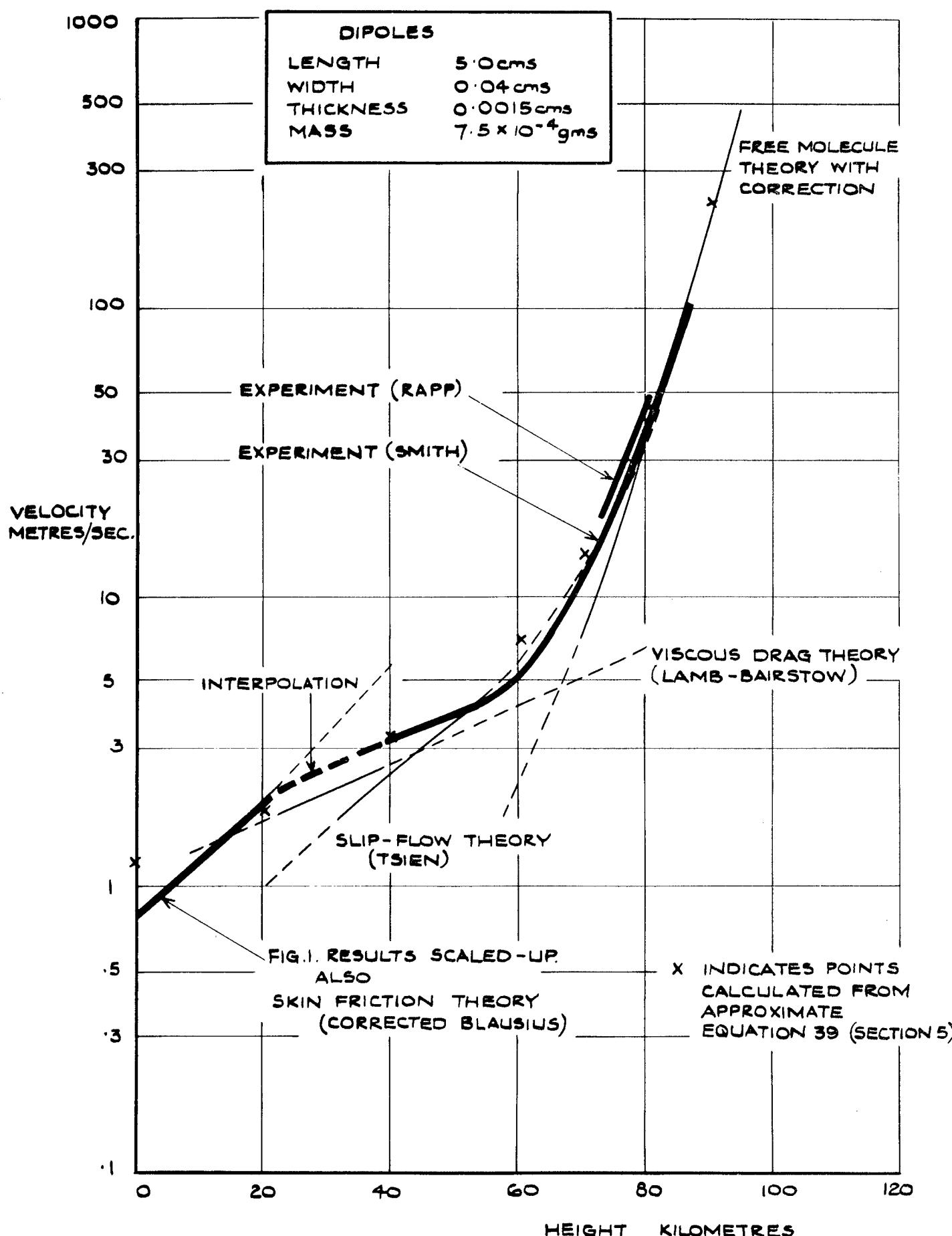


FIG. 3. VELOCITIES AT HIGH ALTITUDES –
BEST THEORETICAL ESTIMATES

Dawson, T.W.G.

531.52 :
 621.372.4 :
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For the most accurate measurements of wind structure it is desirable to employ dipoles with the smallest possible terminal velocity. This means that the dipoles should have low density and the smallest possible thickness.

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